Probabilistic Actual Causation

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Introduction

Type (Generic) Causation:

Asbestos exposure causes mesothelioma.

Actual (Token) Causation:

Mr. Fairchild's exposure to asbestos caused his mesothelioma.

Introduction

Egs:

- K-Pg Extinction
- Cosmic Microwave Background
- Collapse of Bridge 9340 on I-35W
- Financial Crisis
- Outbreak of H7N9 avian 'flu virus

Introduction

Probabilities in science:

- **Quantum Mechanics** (Orthodox, GRW, etc.)
- **Bohmian Mechanics** (Prob. dist. over particle positions)
- **Statistical Mechanics** (Classical or Quantum)
- **High-Level Sciences** (Ecology, Meteorology, Genetics, Chemistry)

Probability-Raising

c, e = events

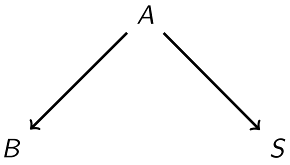
C, E = binary variables

$C = 1$ if c occurs, $C = 0$ otherwise

$E = 1$ if e occurs, $E = 0$ otherwise

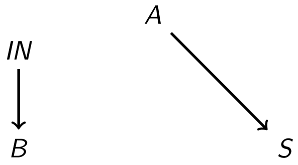
$$P(E = 1|C = 1) > P(E = 1|C = 0)$$

Probability-Raising



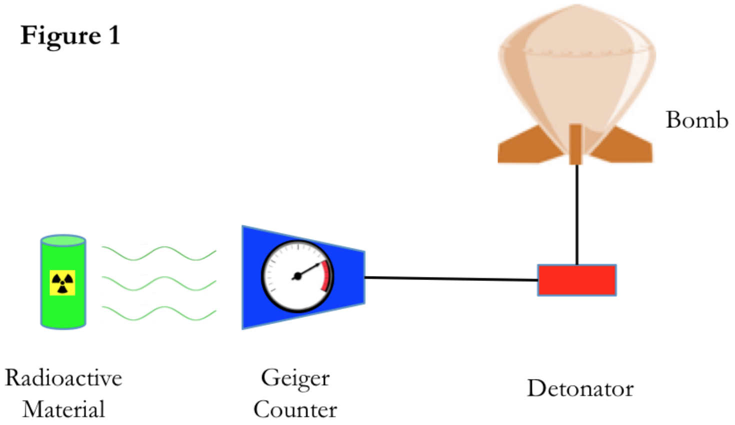
$$P(S = 1|B = 1) > P(S = 1|B = 0)$$

Probability-Raising



$$P(S = 1|do(B = 1)) = P(S = 1|do(B = 0))$$

Three Scenarios



Three Scenarios

Scenarios 1 & 2:

$$P(E = 1|do(M = 1)) > P(E = 1|do(M = 0))$$

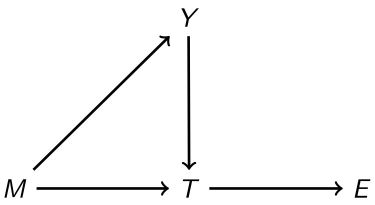
Scenario 3:

$$P(E = 1|do(M = 1)) < P(E = 1|do(M = 0))$$

Three Scenarios

Scenarios 1 & 2:

Scenario 3



Three Scenarios

Scenario 1:

$$P(E=1|do(M=1\&T=1)) > P(E=1|do(M=0))$$

Scenario 2:

$$P(E=1|do(M=1\&T=0)) \leqslant P(E=1|do(M=0))$$

Three Scenarios

Scenario 3:

$$P(E = 1|do(M = 1 \& Y = 0)) > P(E = 1|do(M = 0 \& Y = 0))$$

$$P(E = 1|do(M = 1 \& T = 1 \& Y = 0)) > P(E = 1|do(M = 0 \& Y = 0))$$

Probabilistic Causal Models

Probabilistic causal model: $\mathcal{M} = \langle \mathcal{V}, do(\cdot) \rangle$

$\mathcal{V}$: a set of variables

$V = v$ for $V \in \mathcal{V}$ is a primitive event

$\mathcal{V}$ generates field of events: Boolean closure of set of primitive events.

$do(\cdot)$: function from (conjunctions of) primitive events, $\vec{V} = \vec{v}$, to prob. dists. of form $P(\cdot|do(\vec{V} = \vec{v}))$ – prob. dist. that would result from intervening upon $\vec{V}$ to set $\vec{V} = \vec{v}$

Probabilistic Causal Models

Graphical representation of a probabilistic causal model:

- Variables in $\mathcal{V}$ are nodes
- Directed edge ('arrow') from X to Y ($X,\ Y \in \mathcal{V}$) iff there is
  - some possible assignment of values $\vec{S} = \vec{s}$ to the variables in $\vec{S} = \mathcal{V} \backslash X, Y$;
  - some pair of possible values x, x' of X; &
  - some possible value y of Y

s.t.

$$P(Y = y | do(X = x \& \vec{S} = \vec{s})) \neq P(Y = y | do(X = x' \& \vec{S} = \vec{s}))$$

Path-Specific Probability-Raising

Actual Causation (Simpliciter)

$X = x$ rather than $X = x'$ is an actual cause of $Y = y$ iff $X = x$ & $Y = y$ are the actual values of X & Y and $X = x$ rather than $X = x'$ is an actual cause of $Y = y$ relative to an appropriate model $\mathcal{M}$.

Robust Path-Specific Probability-Raising

Actual Causation (Model-Relative)

$X = x$ rather than $X = x'$ is an actual cause of $Y = y$ relative to a model $\mathcal{M}$ iff there is a path $\mathcal{P}$ in $\mathcal{M}$ s.t., when we hold all variables in $\vec{W} = \mathcal{V} \backslash \mathcal{P}$ fixed at their actual values $\vec{W} = \vec{w}^*$, the probability of $Y = y$ would be higher if $X = x$ than if $X = x'$ even if an arbitrary subset $\vec{Z}'$ of the variables in $\vec{Z} = \mathcal{P} \backslash X, Y$ had taken their actual values, $\vec{Z}' = \vec{z}^*$: formally,

$$P(Y = y | do(X = x \& \vec{Z}' = \vec{z}^* \& \vec{W} = \vec{w}^*))$$

$$>$$

$$P(Y = y | do(X = x' \& \vec{W} = \vec{w}^*))$$

Appropriate Models

What makes a model $\mathcal{M} = \langle \mathcal{V}, do(\cdot) \rangle$ appropriate for assessing whether $X = x$ (rather than $X = x'$) is an actual cause of $Y = y$ (for $X, Y \in \mathcal{V}$)?

1. Prob. dists. of form $P(\cdot | do(\vec{V} = \vec{v}))$ that are the output of $do(\cdot)$ when $\vec{V} = \vec{v}$ is the input must be the 'true' prob. (objective chance?) dist. that would result from intervening upon $\vec{V}$ to set $\vec{V} = \vec{v}$.
2. No two different variables $V_i, V_j \in \mathcal{V}$ should have possible values $V_i = v_i, V_j = v_j$ that represent states of affairs that are logically/metaphysically related.
3. The values of each variable should form a partition.

Appropriate Models

$$M \longrightarrow E$$

$$P(E = 1|do(M = 1)) > P(E = 1|do(M = 0))$$

Appropriate Models

4. If $X = x$ (rather than $X = x'$) is an actual cause of $Y = y$ relative to $\mathcal{M}$, there is no richer model (i.e. no model $\mathcal{M}' = \langle \mathcal{V}', do(\cdots) \rangle$ s.t. $\mathcal{V} \subset \mathcal{V}'$) satisfying 1–3 relative to which $X = x$ (rather than $X = x'$) is not an actual cause of $Y = y$.

Conclusion

Actual causation consists in there being at least one apt model relative to which there is robust path-specific probability-raising.