Probabilistic Actual Causation

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Introduction

Type (Generic) Causation:

Asbestos exposure causes mesothelioma.

Actual (Token) Causation:

Mr. Fairchild’s exposure to asbestos caused his mesothelioma.
Introduction

Egs:

- K-Pg Extinction
- Cosmic Microwave Background
- Collapse of Bridge 9340 on I-35W
- Financial Crisis
- Outbreak of H7N9 avian 'flu virus
Introduction

Probabilities in science:

- **Quantum Mechanics** *(Orthodox, GRW, etc.)*

- **Bohmian Mechanics** *(Prob. dist. over particle positions)*

- **Statistical Mechanics** *(Classical or Quantum)*

- **High-Level Sciences** *(Ecology, Meteorology, Genetics, Chemistry)*
Probability-Raising

c, e = events

C, E = binary variables

C = 1 if c occurs, C = 0 otherwise

E = 1 if e occurs, E = 0 otherwise

P(E = 1|C = 1) > P(E = 1|C = 0)
Probability-Raising

\[ P(S = 1 | B = 1) > P(S = 1 | B = 0) \]
Probability-Raising

$$P(S = 1|do(B = 1)) = P(S = 1|do(B = 0))$$
Three Scenarios

Figure 1

Radioactive Material

Geiger Counter

Detonator

Bomb
Three Scenarios

Scenarios 1 & 2:

\[ P(E = 1|\text{do}(M = 1)) > P(E = 1|\text{do}(M = 0)) \]

Scenario 3:

\[ P(E = 1|\text{do}(M = 1)) < P(E = 1|\text{do}(M = 0)) \]
Three Scenarios

Scenarios 1 & 2:

\[ M \rightarrow T \rightarrow E \]

Scenario 3

\[ Y \]

\[ M \rightarrow T \rightarrow E \]
Three Scenarios

Scenario 1:

\[ P(E = 1|do(M = 1 & T = 1)) > P(E = 1|do(M = 0)) \]

Scenario 2:

\[ P(E = 1|do(M = 1 & T = 0)) \leq P(E = 1|do(M = 0)) \]
Three Scenarios

Scenario 3:

\[ P(E = 1 | do(M = 1 & Y = 0)) > P(E = 1 | do(M = 0 & Y = 0)) \]

\[ P(E = 1 | do(M = 1 & T = 1 & Y = 0)) > P(E = 1 | do(M = 0 & Y = 0)) \]
Probabilistic Causal Models

Probabilistic causal model: $\mathcal{M} = \langle \mathcal{V}, do(\cdot) \rangle$

$\mathcal{V}$: a set of variables

$\mathcal{V} = v$ for $V \in \mathcal{V}$ is a primitive event

$\mathcal{V}$ generates field of events: Boolean closure of set of primitive events.

$do(\cdot)$: function from (conjunctions of) primitive events, $\vec{V} = \vec{v}$, to prob. dists. of form $P(\cdot | do(\vec{V} = \vec{v}))$ – prob. dist. that would result from intervening upon $\vec{V}$ to set $\vec{V} = \vec{v}$
Probabilistic Causal Models

Graphical representation of a probabilistic causal model:

- Variables in $\mathcal{V}$ are nodes
- Directed edge (‘arrow’) from $X$ to $Y$ ($X, Y \in \mathcal{V}$) iff there is
  - some possible assignment of values $\tilde{S} = \tilde{s}$ to the variables in $\tilde{S} = \mathcal{V} \setminus X, Y$;
  - some pair of possible values $x, x'$ of $X$; &
  - some possible value $y$ of $Y$

s.t.

$$P(Y = y | do(X = x & \tilde{S} = \tilde{s})) \neq P(Y = y | do(X = x' & \tilde{S} = \tilde{s}))$$
Path-Specific Probability-Raising

Actual Causation (Simpliciter)

\( X = x \text{ rather than } X = x' \) is an actual cause of \( Y = y \) iff \( X = x \text{ & } Y = y \) are the actual values of \( X \text{ & } Y \) and \( X = x \text{ rather than } X = x' \) is an actual cause of \( Y = y \) relative to an appropriate model \( M \).
Robust Path-Specific Probability-Raising

Actual Causation (Model-Relative)

\[ X = x \] rather than \( X = x' \) is an actual cause of \( Y = y \) relative to a model \( \mathcal{M} \) iff there is a path \( \mathcal{P} \) in \( \mathcal{M} \) s.t., when we hold all variables in \( \vec{W} = \mathcal{V} \setminus \mathcal{P} \) fixed at their actual values \( \vec{W} = \vec{w}^* \), the probability of \( Y = y \) would be higher if \( X = x \) than if \( X = x' \) even if an arbitrary subset \( \vec{Z}' \) of the variables in \( \vec{Z} = \mathcal{P} \setminus X \), \( Y \) had taken their actual values, \( \vec{Z}' = \vec{z}^* \): formally,

\[
P(Y = y \mid do(X = x & \vec{Z}' = \vec{z}^* & \vec{W} = \vec{w}^*)) > P(Y = y \mid do(X = x' & \vec{W} = \vec{w}^*))
\]
Appropriate Models

What makes a model $\mathcal{M} = \langle \mathcal{V}, do(\cdot) \rangle$ appropriate for assessing whether $X = x$ (rather than $X = x'$) is an actual cause of $Y = y$ (for $X, Y \in \mathcal{V}$)?

1. Prob. dists. of form $P(\cdot | do(\vec{V} = \vec{v}))$ that are the output of $do(\cdot)$ when $\vec{V} = \vec{v}$ is the input must be the ‘true’ prob. (objective chance?) dist. that would result from intervening upon $\vec{V}$ to set $\vec{V} = \vec{v}$.

2. No two different variables $V_i, V_j \in \mathcal{V}$ should have possible values $V_i = v_i, V_j = v_j$ that represent states of affairs that are logically/metaphysically related.

3. The values of each variable should form a partition.
Appropriate Models

\[ M \rightarrow E \]

\[ P(E = 1|do(M = 1)) > P(E = 1|do(M = 0)) \]
4. If $X = x$ (rather than $X = x'$) is an actual cause of $Y = y$ relative to $\mathcal{M}$, there is no richer model (i.e. no model $\mathcal{M}' = \langle \mathcal{V}', \text{do}(\cdots) \rangle$ s.t. $\mathcal{V} \subset \mathcal{V}'$) satisfying 1–3 relative to which $X = x$ (rather than $X = x'$) is not an actual cause of $Y = y$. 
Actual causation consists in there being at least one apt model relative to which there is robust path-specific probability-raising.